

Summer School for **Mathematics Education** Monday 16 – Friday 27 August 2010

Utrecht University
Utrecht School of Applied Sciences
Course number H7

Information about the program and the content of the course

AIM OF THE COURSE

Bringing participants up-to-date in curriculum development and research in the field of (science and) mathematics education. Refreshing and deepening the knowledge in core subjects of various fields of Science and mathematics.

The programme for mathematics education you will find on page 5, for science education on page 11.

Lecturers and workshop leaders:

1. Prof. dr. Harrie Eijkelhof
Director of the Summer Course
Vice dean of the Faculty of Science
2. Prof. dr. Jan van Maanen
Director Math section of the Freudenthal Institute for Science and Mathematics
Education
3. Drs. Aldine Aaten
Phd-student
4. Dr. Barbara van Amerom
Staff member
5. Dr. Arthur Bakker
Staff member
6. Dr. Dirk-Jan Boerwinkel
Staff member
7. Drs. Peter Boon
Staff member
8. Dr. Peter Dekkers
Staff member
9. Dr. Paul Drijvers
Staff member
10. Dr. Michiel Doorman
Staff member
11. Mieke Abels
Staff member
12. Drs. Aad Goddijn
Staff member
13. Mevr. drs. Dédé de Haan
Staff member
14. Dr. Jaap den Hertog
Staff member
Director of the Summer Course
15. Martin Kindt
Retired Staff member
16. Ir. Henk van der Kooij
Staff member

17. Dr. Koos Kortland
Staff member
18. Dr. Jo Nelissen
Staff member
19. Prof. dr. Albert Pilot
Director of the Freudenthal Institute for Science and Mathematics Education
20. Dr. Gjalt Prins
Staff member
21. Mevr. Sietske Tacoma (BAS)
22. Dr. Ton van der Valk
Staff member
23. Mevr. Drs. Monica Wijers
Staff member

Participants Summer School Mathematics Education 2010

Rafael Barcellano Buemia	Philippines
Michael Christopher Davis	United States of America
Damla AyŞe Geçim	Turkey
Mehmet Fatih Ocal	Turkey
Tugba Yalcin	Turkey
Bilge Yurekli	Turkey
Maria Zamora	Spain
Lawrence Egyir	Ghana
Shabir Akhtar	Pakistan
Cut Khairunnisak	Indonesia
Veronika Fitri Rianasar	Indonesia
Fatim Khikmiyah	Indonesia
Nenden Octavarulia Shanty	Indonesia
Elisabet Permata Sari	Indonesia
Zetra Putra	Indonesia
Fridgo Tasman	Indonesia
Yeka Meryansumayeka	Indonesia
Puri Pramudiani	Indonesia
Shintia Revina	Indonesia
Destina Winarti	Indonesia
Nia Yberta	Indonesia
Latif Anwar	Indonesia

Participants Summer School Science Education 2010

Paulius Lukas Tamosiunas	Lithuania
İlkay Buket Ataç Özdemir	Turkey
Diber Demirtaş	Turkey
Fatma Caner	Turkey
Nilay Öztürk	Turkey
Jeanne Kriek	South Africa

Programme Summer School mathematics education “Making mathematics meaningful for students”

Program

Monday August 16th (combination with the participants of science)

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|---------------|---|
| 9.00 – 9.30 | Intake for all participants (Location Freudenthal Institute, Aïdadreef 12, Utrecht) |
| 9.30 – 9.45 | Opening and welcome by dr. Jaap den Hertog, Director of the Summer School |
| 9.45 – 10.30 | Lecture by dr. Jaap den Hertog
“Introduction to the Dutch school system and to RME” |
| 10.30 – 10.45 | Coffee, tea, etc. |
| 10.45 – 12.30 | Workshop by Prof. dr. Albert Pilot
“Design research” |
| 12.30 – 13.30 | Lunch |
| 13.30 – 14.00 | Lecture by Prof. dr. Harrie Eijkelhof
“Innovations in Dutch science and mathematics education” |
| 14.00 – 14.30 | Lecture by Douglas D. Agyei (participant of the Summer School 2009)
“Developing Technological Pedagogical and Content Knowledge Through Teacher Design Teams: The Case of Mathematics Teachers in Ghana” |
| 15.00 – 15.15 | Coffee, tea, etc. |
| 15.15 – 17.30 | Welcome to Utrecht |

Welcome to Utrecht! In groups of approximately 15 students you will explore the city. A Dutch student will lead the way. You will see all kinds of interesting places as you walk alongside the canals through the historical city center, such as the Dom Tower, the place where we will have a drink each week, and so on. The walk will end by movie theater 't Hoogt, where every Monday students attending Utrecht Summer School can watch a Dutch movie for free!

Thursday August 19th

- 9.00 – 9.30 Lecture by Monica Wijers
 “Open-ended modeling tasks in Dutch math education”
- 9.30 – 12.30 Problem solving workshop by Dédé de Haan and Monica Wijers
- 10.30– 11.00 Coffee, tea, etc.
- 12.30 – 13.30 Lunch
- 14.30 – 17.00 Workshop by Monica Wijers and Dede de Haan
 “Assessment of large problem solving tasks”
- 17.00 – 18.00 The Faculty of Science offers a drink in the restaurant of the Minnaert Building.

Friday August 20th

- 9.00 – 12.30 Workshop by Mieke Abels
 “The iceberg-metaphor and applications”

Combination with the participants of science:

- 12.30 – 13.30 Lunch
- 14.00 – 17.00 Exchange between the participants of the science and of the mathematical stream of this Summer School

Saturday August 21st

Free time for shopping and town visit

Sunday August 22nd

Excursion (not obligatory)

Monday 23rd

- 9.00 – 10.00 Workshop by dr. Jaap den Hertog
“Fractions, a problem solving approach”
- 11.00 – 11.30 Coffee, tea, etc.
- 10.00 – 12.30 Workshop by Henk van der Kooij
“Logarithms: a common sense approach”
- 12.30 – 13.30 Lunch
- 13.30 – 14.30 Workshop by Henk van der Kooij, part II
- 14.30 – 17.00 Time for doing tasks and presentations by the participants

Tuesday 24th

- 9.00 – 11.00 Workshop by dr. Michiel Doorman and dr. Paul Drijvers
“Supporting Functional Thinking”
- 11.00 – 11.30 Coffee, tea, etc.
- 11.30 – 12.30 Presentation by dr. Arthur Bakker
“Learning to compute concentrations with a computer tool in
laboratory education”
- 12.30 – 13.30 Lunch
- 13.30 – 14.30 Workshop by dr. Michiel Doorman and dr. Paul Drijvers, part II
- 14.30 – 17.00 Time for doing tasks and presentations by the participants
- 18.30 – 21.30 Dinner Summer School

Wednesday 25th

- 9.00 – 11.00 Workshop by Aldine Aaten
“Picture books for Kindergarten and mathematical thinking”
- 11.00 – 11.30 Coffee, tea, etc.
- 11.30 – 12.30 Presentation by dr. Jaap den Hertog
“The phenomenological approach of Hans Freudenthal”
- 12.30 – 13.30 Lunch
- 13.30 – 14.30 Workshop by Aldine Aaten, part II
- 14.30 – 17.00 Time for doing tasks and presentations by the participants

Thursday 26th

- 9.00 – 11.00 Workshop by Aad Goddijn and Sietske Tacoma
“Linear functions of complex numbers”
- 11.00 – 11.30 Coffee, tea, etc.
- 11.30 – 12.30 Workshop by Peter Boon
“The Digital Mathematics Environment (DME)”
- 12.30 – 13.30 Lunch
- 13.30 – 14,30 Workshop by Peter Boon, part II
- 14.30 – 17.00 Time for doing tasks and presentations by the participants

Friday 27th (combination with the participants of science)

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| 9.00 – 9.30 | Lecture by Mieke Abels
“The Assessment Pyramid” |
| 9.30 – 14.00 | Workshop by Mieke Abels and Jaap den Hertog
“About Assessment” The focus of this workshop is on the analysis of a balanced assessment, the design of an assessment problem, and if time permits, the design of a balanced assessment. |
| 10.30– 11.00 | Coffee, tea, etc. |
| 12.30 – 13.30 | Lunch |
| 14.30 – 16.00 | Presentations by the participants; evaluation of the Summer School |
| 16.00 – 17.00 | Happy hour |

Programme summer school science education

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10.30 – 10.45	Coffee, tea, etc.
10.45 – 12.30	Workshop by Prof. dr. Albert Pilot “Design research”
12.30 – 13.30	Lunch
13.30 – 14.00	Lecture by Prof. dr. Harrie Eijkelhof “Innovations in Dutch science and mathematics education”
14.00 – 14.30	Lecture by Douglas D. Agyei (participant of the Summer School 2009) “Developing Technological Pedagogical and Content Knowledge Through Teacher Design Teams: The Case of Mathematics Teachers in Ghana”
15.00 – 15.15	Coffee, tea, etc.
15.15 – 17.30	Welcome to Utrecht

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Tuesday August 17th

Morning (Harrie Eijkelhof et al):

- introduction to the science programme,
- inventory of expectations of participants

Wednesday August 18th

Morning (Peter Dekkers): Developments in primary science education

Afternoon (Peter Dekkers): Visit to the Universiteitsmuseum

Thursday August 19th

Morning (Harrie Eijkelhof): Comparing science education in various countries (PISA, TIMSS)

Afternoon: Presentations by participants

Friday August 20th

Morning (Koos Kortland): Science practicals for secondary students

Afternoon: Exchange between the participants of the science and of the mathematical stream of this Summer School

Saturday August 21^{rst}

Free time for shopping and town visit

Sunday August 22nd

Excursion (not obligatory)

Monday 23rd

All day (Harrie Eijkelhof, Peter Dekkers, Koos Kortland, Gjalt Prins): Curriculum development in science education in NL

Tuesday 24th

All day (Dirk Jan Boerwinkel: Form and function in primary science education

Dinner (all lecturers invited)

Wednesday 25th

All day (various lecturers): Examples of PhD-studies in science education

Thursday 26th

All day (various lecturers): Advanced topics in science education (preference of participants)

Friday 27th

9.00 – 9.30	Lecture by Mieke Abels “The Assessment Pyramid”
9.30 – 14.00	Workshop by Mieke Abels and Jaap den Hertog About Assessment.
10.30– 11.00	Coffee, tea, etc.
12.30 – 13.30	Lunch
14.30 – 16.00	Presentations by the participants; evaluation of the Summer School
16.00 – 17.00	Happy hour

Descriptions of the lectures and workshops

Albert Pilot: The method of Educational Design-Based Research: why and how?

Educational Design-Based Research (EDBR) is a cyclic research method for studying design principles and the synthesis of such principles into design frameworks. In EDBR a first design of a teaching and learning sequence or unit is based on theoretical aspects and experiences, leading to a synthesis of design principles into a first framework for designing. The detailed design of the domain specific teaching and learning sequence is accompanied with a set of argued expectations how a sequence or unit is functioning, and why it should operate according to these expectations. The aim is a better understanding of the teaching and learning sequence, the framework for designing and the learning results of the students.

Douglas D. Agyei: “Developing Technological Pedagogical and Content Knowledge Through Teacher Design Teams: The Case of Mathematics Teachers in Ghana”

Although many studies have shown the need to better align teachers’ preparation in the integration of technology in classroom practice, most teachers in Ghana have not had any preparation that develops their Technology Pedagogical and Content Knowledge (TPACK). This paper presents a case study of four mathematics teachers at the University of Cape Coast, Ghana; who worked in two design teams to develop lessons and subsequently taught in a technology-based environment for the first time. To make them familiar with technology use in mathematics teaching the teachers first discussed and used exemplary lesson materials, which were prepared by the researcher. Based on this experience the teachers designed their own technology-based lessons, which they taught to a group of teachers. Interview, observation, and survey data were collected throughout the study. The study revealed key attributes of a professional development scenario to foster growth of teachers in TPACK.

Barbara van Amerom: Number and algebra

Students in secondary schools need to master algebraic skills. To do so, they need to do practice a lot, but practice without understanding isn't very effective. A proper balance is needed to make algebra meaningful. This means that students also have to develop algebraic thinking. Within the topic of number we see good opportunities to do so. In this workshop we will look at ways that students can practice number and algebra in a way that is meaningful to them. We will also analyse some student work and discuss the results and the implications for the teaching and learning of algebra.

Martin Kindt: A natural way to Algebra

In the algebra chapters of present-day Math textbooks in the Netherlands there is a certain overkill of graphs, tables and formulas, at the expense of training in algebraic skills. Anyway, that’s the opinion of a lot of professional mathematicians. In my opinion, these criticasters are not totally wrong. Undoubtedly the ‘functional approach’ of algebra is very important, but

there is another valuable entrance of algebra. Students are familiar with natural numbers, so may be the best start of algebra could be one which handles about problems and patterns situated in this world. There are nice historical points of contact, for instance in the work of the Pythagoreans and of Nicomachos. In this session we will discuss some challenging problems in the domain which sometimes is called: 'patterns and figures'.

Jan van Maanen: History in mathematics education with an emphasis on the teaching and learning of geometry.

In this workshop you will experience the phases in integrating history in mathematics lessons, in order to better teach the mathematical contents. Reading old mathematics will be part of the programme, but also studying texts which teach a mathematical subject through a historical 'lense'.

As a preparation you could try to find the ICMI Study "History in Mathematics Education", edited by John Fauvel and Jan van Maanen (Dordrecht: Kluwer Academic Publishers, 2000). Attached is the introductory chapter, and an article about the principle of integrating historical elements in mathematics teaching ("New maths may profit from old methods").

Furthermore, you could consult the article Iris Gulikers and Klaske Blok, 'A historical angle', a survey of recent literature on the use and value of history in geometrical education in: Educational studies in mathematics, Volume: 47, Issue: 2 (2001), pp: 223-258 (electonical version available in: <http://www.springerlink.com/content/p6u7yf8n6xcku5qg/fulltext.pdf>)

Jo Nelissen: Mathematics and intuition

The concept of intuition has always drawn attention and stimulated the imagination, but evoked a lot of doubt and criti-cism as well. In this article, the idea is defended that there is no point in creating a contradiction between intuition and cognition, because this contradiction is counterproductive. Intuition is considered a human activity that is closely con-nected with knowledge and that emerges during problem solving. We discern and discuss two levels of intuition, and the process of intuitive thinking is characterized in psychological terms as an immediate grasping of the meaning of a struc-ture or Gestalt. Intuition may support, as well as thwart, the learning process. In mathematics education daily life based experience (common sense) is in most cases a good starting point for the teaching process, but it also can happen that intuition - based on common sense - works towards the acquisition of mathematical insights.

The concept of intuition plays a central role in the 'intuitionism' of Brouwer. In Brouwer's view, axioms are seen as in-complete, and this means that the search for valid and true knowledge cannot be viewed as just a game of logical rules, but as a never ending human activity. Intuition can and should be supported in interactive mathematical education, because mathematics is not just a case of learning to apply standard rules and procedures. Students should be encouraged to explore unknown and unfamiliar problems. Hence, intuition is connected, not just with cognition and knowledge, but also with daring the unfamiliar, emotion and creativity.

Monica Wijers and Dédé de Haan: Open-ended modeling tasks in Dutch math education; The mathematics Alympiad: working in groups on an open-ended modelling task (<http://www.fi.uu.nl/alympiade/en/>)

At all levels of mathematics education in the Netherlands large open ended modelling tasks - to be completed by groups of students- have come to stay. In this whole-day workshop we will first present an overview of this type of tasks in Dutch math education.

Next we will zoom in on an example for upper secondary schools. Since 1989 an annual modeling competition is organized in upper secondary schools, offering an open-ended modeling assignment, which is often derived from an authentic real-life problem related to the social and economical sciences, with an emphasis on discrete mathematics and statistics (as described in 'Backgrounds').

The competition for teams of students was initiated because of growing discomfort with the limitations of written tests, which did not enable students to engage in a full modeling cycle. The annual Math A-lympiad has two rounds: the first round is the preliminary, which takes place in the participating schools. Here the modelling task has to be completed within a day (a Friday in November). The second round is the final, in which the twelve best teams from the first round compete in a central location. The final task is harder than the preliminary task, and had to be completed within two days (a Friday and Saturday in April).

Participants of this workshop will work on a part of this assignment in pairs. The results will be presented and in the last part of this day we will discuss ways to judge the quality of student work on these assignments.

Mieke Abels: The iceberg-metaphor and applications

Researchers at the Freudenthal Institute developed the iceberg model to support teacher thinking about learning processes and strategies used by students (Boswinkel and Moerlands, 2001). This model has proved to be a powerful metaphor for illustrating how students need to experience a broad range of mathematical models to make sense of formal mathematical representations. The construction of an iceberg is a good experience to explore and discuss essential models and representations and strategies which can help students to learn mathematics with understanding. The next step is the development of a sequence of activities, based on the iceberg metaphor.

During this workshop, the participants will do several activities to develop an understanding of the iceberg metaphor, and will investigate a pathway for the concept of area.

Jaap den Hertog: Fractions, a problem solving approach

In many countries, the transition from arithmetic towards algebra poses a problem. Can this gap become be bridged? Which part of the iceberg can be seen? We will discuss examples of a long learning trajectory with fractions, starting at elementary education and ending in secondary education. As a group, we will look for new possibilities and newly designed materials. We hope to solve at least a fraction of the problem!

Aldine Aaten: Picture books and mathematical thinking in kindergarten

Improving children's mathematical understanding at a young age is beneficial for their later mathematical development. The PICO-ma project (Picture Books and Concept Development in Mathematics) aims to add to the knowledge on teaching mathematics to young children by investigating the effect of reading picture books to kindergartners on their mathematical understanding. The PICO-ma project is carried out by Prof. dr. Marja van den Heuvel-Panhuizen (project leader) and Msc Aldine Aaten (PhD student).

In this workshop, we will discuss what it means to do mathematics at kindergarten level and why picture books are suitable for teaching mathematics to children at this young age. Furthermore, we will investigate some picture books more closely to identify which characteristics of picture books make them useful and powerful for teaching mathematics.

Paul Drijvers and Michiel Doorman: Supporting Functional Thinking

In this session we will discuss key aspects of the function concept and study a recent instructional design for this topic (grade 8; 13-14 years).

The goal of the design is that students understand functions as a model for repeated calculations, representing an input-output relationship. A model that can be used to solve problems about dependency relationships with tables, graphs and formulas. The design starts with contextual problems and the focus shifts to general concepts. We will reflect on the tenets of RME (e.g. emergent modeling) with respect to this topic, the design and design research including quantitative as well as qualitative methods.

More information about the research project can be found through:

<http://www.fi.uu.nl/tooluse/en/>

Arthur Bakker: Learning to compute concentrations with a computer tool in laboratory education

The presentation concerns highlights of research carried out in secondary vocational education. Because many students in laboratory education find computations around dilution and concentrations of chemical substances hard, we have designed a computer tool to help them. Participants will get the opportunity to work with this tool during this time slot.

Henk van der Kooij: "Logarithms: a common sense approach"

Logarithms and logarithmic functions are seen as one of the most difficult mathematical topics in senior secondary education. In most countries the logarithmic function is introduced as the inverse of the exponential function: $y = a^x \Leftrightarrow x = \log_a y$. In the Netherlands, a common sense approach is used. A thorough study of characteristics of exponential growth processes leads in a natural way to a deep understanding of logarithms. We will work on and discuss this approach during the workshop.

Aad Goddijn en Sietske Tacoma: "linear functions of complex numbers"

Complex numbers can be approached and taught in many different ways. In this workshop we will focus on a geometric approach to complex numbers. Linear functions play an important role in dealing with complex numbers geometrically. Therefore, we will study linear functions of complex numbers. We will see their use in the domain of complex numbers, but we will also see ways in which these functions can lead to elegant proofs of Euclidean, non-complex, geometrical problems.

Peter Boon: The Digital Mathematics Environment (DME)

The DME is a web-based electronic learning environment developed by the Freudenthal Institute. It provides students' access to interactive and dynamic tasks within many mathematical domains. These tasks, implemented in Java-applets, are small interactive pedagogical programs, which students can use for several purposes, such as to explore a problem situation, to discover a representation or a concept, to construct and explore 3dobjects, to carry out a task or to practice a skill.

Embedding the applets in the DME offers several possibilities:

1. They can be organized in sequences that can be made available to groups of students or classes by the teacher.
2. The DME allows for storing the students' work with the applets.
3. The DME offers the teacher access to the students' work, by means of all-class result overviews and in detail as well.
4. The ready-made tasks in applets can be configured by the teacher. New items within the applet task can be added and existing items can, for example be adjusted to the level of the students.

A new and powerful option of the DME is a mathematical editor (authoring tool) for designing completely new tasks for students. The editor has a user-friendly formula-editor and a powerful formula parser to handle the answers of students. It offers the possibility to provide (intermediate) feedback within the solving process of the students. It is also possible to provide several tools and interactive models that support the learning process.

The DME is used by more than 100 schools in the Netherlands. The authoring tool is also used by the main textbook publishers. Development takes place in close cooperation with teachers. Meanwhile, research is conducted to theoretically underpin the value of using DME in teaching practice.

In this workshop I will show the different options and the design principles that played a role in the development of the DME and discuss them with you. In the second part of the workshop we will let you work with the DME to enable you to experience the different possibilities.

Participants of the hands-on workshop can have a look at ICT-rich, interactive learning materials and at features of the Digital Mathematics Environment, focused on space geometry. The activities will show that the applets can bridge the gap between concrete and abstract experiences.

A preview: <http://www.fi.uu.nl/dwo/en>

Mieke Abels: The Assessment Pyramid

In a balanced test, students should have the opportunity to show what they have learned and what they are able to do. Teachers may want to describe student understanding and capabilities. Also, teachers may want to describe students' misconceptions, partial understanding, etc. They want answers to questions about students' mathematical abilities, such as:

- does a student only master basic skills?
- is he able to give proper (mathematical) reasons?
- can he solve more complex problems?

Because a teacher wants to give the student feedback, (s)he needs to know which subjects are not mastered yet by the students and perhaps need more emphasis in class. It is also important to know which steps must be taken to help the least able students and give the more gifted ones enough challenging materials.

To find answers for all of these questions teachers need a great variety of tasks and tests on different levels of competency. Moreover the mathematical content of the problem and the complexity of the task must be considered.

As you see, there are many factors to be taken into account! Help is needed to bring order to the chaos. We will show a model for the classification of problems known as 'Jan de Lange's pyramid'.