

# Summer School for **Mathematics Education** Monday 15 – Friday 26 August 2011

Utrecht University  
Utrecht School of Applied Sciences  
Course number H7

## Information about the program and the content of the course

### AIM OF THE COURSE

This course is targeted at science and mathematics teachers with a strong interest in educational research and curriculum development, and is especially suited for those aspiring to do a PhD project. The course provides an introduction to current trends in curriculum development and research in science and mathematics education. The perspective is international, with some special attention for current developments in the Dutch educational system, such as design research, context-based science and mathematics education, and fostering talent.

## Lecturers and workshop leaders:

1. Prof. dr. Harrie Eijkelhof  
Director of the Freudenthal Institute for Science and Mathematics
2. Prof. dr. Jan van Maanen  
Director Math section of the Freudenthal Institute for Science and Mathematics  
Education
3. Dr. Barbara van Amerom  
Staff member
4. Dr. Arthur Bakker  
Staff member
5. Drs. Peter Boon  
Staff member
6. Dr. Paul Drijvers  
Staff member
7. Dr. Michiel Doorman  
Staff member
8. Mieke Abels  
Staff member
9. Drs. Aad Goddijn  
Staff member
10. Drs. Dédé de Haan  
Staff member
11. Dr. Jaap den Hertog  
Staff member  
Director of the Summer Course
12. Dr. Dolly van Eerde  
Staff member
13. Martin Kindt  
Retired Staff member
14. Ir. Henk van der Kooij  
Staff member
15. Dr. Jo Nelissen  
Staff member
16. Dr. Elwin Savelsbergh  
Staff member  
Director of the Summer Course
17. Drs. Monica Wijers  
Staff member

## Participants Summer School Mathematics Education 2011

1. Belen Garcia Spain
2. Colin Jackson United Kingdom
3. Caroline Kuhn Spain
4. Ekaterina Miroshnichenko Russia
5. Gert Hendrikus Stols South Africa
6. Yenny Anggreini Sarumaha Indonesië
7. Ekasatya Aldila Afriansyah Indonesië
8. Saliza Safta Assiti Indonesië
9. Ilham Rizkianto Indonesië
10. Septy Sari Yukans Indonesië
11. Anisa Fatwa Sari Indonesië
12. Achmad Badrun Kurnia Indonesië
13. Nila Mareta Murdiyani Indonesië
14. Fajar Arwadi Indonesië

# Programme Summer School mathematics education “Making mathematics meaningful for students”

## Program

### Monday August 15<sup>th</sup>

9.00 – 9.30	Intake
9.30 – 9.45	Opening and welcome by Jaap den Hertog, Director of the Summer School
9.45 – 10.30	Lecture by Jaap den Hertog “Introduction to the Dutch school system and to RME”
10.30 – 10.45	Coffee, tea, etc.
10.45 – 12.30	Workshop by Harrie Eijkelhof “International comparisons of science and mathematics education: TIMSS and PISA”
12.30 – 13.30	Lunch
13.30 – 16.00	Workshop by Harrie Eijkelhof, part II
15.00 – 15.15	Coffee, tea, etc.
16.00 – 17.30	Presentations by the participants of the Summer School
18.00 – 22.00	Scavenger Hunt (Costs: 10 euro) ( <a href="http://www.utrechtsummerschool.nl/index.php?type=content&amp;page=socialprogramme&amp;activity=18">http://www.utrechtsummerschool.nl/index.php?type=content&amp;page=socialprogramme&amp;activity=18</a> )

**Tuesday August 16<sup>th</sup>**

9.00 – 11.00	Workshop by Barbara van Amerom “Number and Algebra”
11.00– 11.30	Coffee, tea, etc.
11.30 – 12.30	Presentation by Martin Kindt “A natural way to Algebra”
12.30 – 13.30	Lunch
13.30 – 14.30	Workshop by Barbara van Amerom, part II
14.30 – 17.00	Time for doing tasks and presentations by the participants N.B. There are three possibilities: 1. Performing tasks from the workshops; 2. To design tasks for their own practice; 3. To deepen your own researches (with help of other participants or staff members of the Freudenthal Institute).

**Wednesday August 17<sup>th</sup>**

9.00 – 10.00	Workshop by Jaap den Hertog “Fractions, a problem solving approach”
11.00 – 11.30	Coffee, tea, etc.
10.00 – 12.30	Workshop by Henk van der Kooij “Logarithms: a common sense approach”
12.30 – 13.30	Lunch
13.30 – 14.30	Workshop by Henk van der Kooij, part II
14.30 – 17.00	Time for doing tasks and presentations by the participants

**Thursday August 18<sup>th</sup>**

- 9.00 – 9.30           Lecture by Monica Wijers  
                          “Open-ended modeling tasks in Dutch math education”
- 9.30 – 12.30           Problem solving workshop by Dédé de Haan and Monica Wijers
- 10.30– 11.00           Coffee, tea, etc.
- 12.30 – 13.30           Lunch
- 14.30 – 17.00           Workshop by Monica Wijers and Dédé de Haan  
                          “Assessment of large problem solving tasks”
- 17.00 – 18.00           Summer School Drink from the Graduate School of Natural Sciences in  
the restaurant of the Minnaert Building.

**Friday August 19<sup>th</sup>**

- 9.00 – 11.00           Workshop by Jan van Maanen  
                          “Palpable arithmetic”
- 11.00– 11.30           Coffee, tea, etc.
- 11.30 – 12.30           Presentation by Jo Nelissen  
                          “Mathematics and intuition”
- 12.30 – 13.30           Lunch
- 13.30 -14.30           Workshop by Jan van Maanen, part II
- 14.30 – 17.00           Time for doing tasks and presentations by the participants

**Saturday August 21<sup>rst</sup>**

Free time for shopping and town visit

**Sunday August 22<sup>nd</sup>**

Excursion (not obligatory)

**Monday 22<sup>nd</sup>**

- 9.00 – 12.30      Workshop by Mieke Abels  
   “The iceberg-metaphor and applications”
- 12.30 – 13.30      Lunch
- 14.00 – 17.00      Presentation participants

**Tuesday 23<sup>th</sup>**

- 9.00 – 12.30      Workshop by Aad Goddijn and Elwin Savelsbergh  
   “Modeling dynamic behavior; numerical and analytic  
   approaches”
- 11.00 – 11.30      Coffee, tea, etc.
- 12.30 – 13.30      Lunch
- 13.30 – 14.30      Lecture by Dolly van Eerde  
   “Language in mathematics education”
- 18.30 – 21.30      Dinner Summer School

**Wednesday 24<sup>th</sup>**

- 9.00 – 11.00      School visit
- 12.30 – 13.30      Lunch
- 11.30 – 12.30      Workshop by Peter Boon  
   “The Digital Mathematics Environment (DME)”
- 12.30 – 13.30      Lunch
- 13.30 – 14.30      Workshop by Peter Boon, part II
- 14.30 – 17.00      Time for doing tasks and presentations by the participants



## Descriptions of the lectures and workshops

### **Harrie Eijkelhof: International comparisons of science and mathematics education: TIMSS and PISA**

Over the last two decades several large-scale international studies were carried out to compare students' knowledge, skills and attitudes in science and mathematics education between countries. The TIMSS studies focus on the learning results in science and mathematics at ages 9/10, 13/14 and 17/18. The PISA-programme deals with reading and mathematical and scientific literacy at age 15.

In this session we will explore the goals of these studies, their framework and the research methods used. We will look in detail at various results and the serious implications in some countries. Finally we will discuss the value and limitations of this kind of studies.

### **Barbara van Amerom: Number and algebra**

Students in secondary schools need to master algebraic skills. To do so, they need to practice a lot, but practice without understanding isn't very effective. A proper balance is needed to make algebra meaningful. This means that students also have to develop algebraic thinking. Within the topic of number we see good opportunities to do so.

In this workshop we will look at ways that students can practice number and algebra in a way that is meaningful to them. We will also analyze some student work and discuss the results and the implications for the teaching and learning of algebra.

### **Martin Kindt: A natural way to Algebra**

In the algebra chapters of present-day Math textbooks in the Netherlands there is a certain overkill of graphs, tables and formulas, at the expense of training in algebraic skills. Anyway, that's the opinion of a lot of professional mathematicians. In my opinion, these criticasters are not totally wrong. Undoubtedly the 'functional approach' of algebra is very important, but there is another valuable entrance of algebra. Students are familiar with natural numbers, so may be the best start of algebra could be one which handles about problems and patterns situated in this world. There are nice historical points of contact, for instance in the work of the Pythagoreans and of Nicomachos. In this session we will discuss some challenging problems in the domain which sometimes is called: 'patterns and figures'.

### **Jaap den Hertog: Fractions, a problem solving approach**

In many countries, the transition from arithmetic towards algebra poses a problem. Can this gap become be bridged? Which part of the iceberg can be seen? We will discuss examples of a long learning trajectory with fractions, starting at elementary education and ending in secondary education. As a group, we will look for new possibilities and newly designed materials. We hope to solve at least a fraction of the problem!

### **Henk van der Kooij: "Logarithms: a common sense approach"**

Logarithms and logarithmic functions are seen as one of the most difficult mathematical topics in senior secondary education. In most countries the logarithmic function is introduced as the inverse of the exponential function:  $y = a^x \Leftrightarrow x = \log_a y$ . In the Netherlands, a common sense approach is used. A thorough study of characteristics of exponential growth processes leads in a natural way to a deep understanding of logarithms. We will work on and discuss this approach during the workshop.

**Monica Wijers & Dede De Haan: Open-ended modeling tasks in Dutch math education; The mathematics Olympiad: working in groups on an open-ended modeling task**  
[\(http://www.fi.uu.nl/olympiade/en/\)](http://www.fi.uu.nl/olympiade/en/)

At all levels of mathematics education in the Netherlands large open ended modeling tasks -to be completed by groups of students- have come to stay. In this whole-day workshop we will first present an overview of this type of tasks in Dutch math education.

Next we will zoom in on an example for upper secondary schools. Since 1989 an annual modeling competition is organized in upper secondary schools, offering an open-ended modeling assignment, which is often derived from an authentic real-life problem related to the social and economical sciences, with an emphasis on discrete mathematics and statistics (as described in 'Backgrounds').

The competition for teams of students was initiated because of growing discomfort with the limitations of written tests, which did not enable students to engage in a full modeling cycle. The annual Math A-lympiad has two rounds: the first round is the preliminary, which takes place in the participating schools. Here the modelling task has to be completed within a day (a Friday in November). The second round is the final, in which the twelve best teams from the first round compete in a central location. The final task is harder than the preliminary task, and had to be completed within two days (a Friday and Saturday in April).

Participants of this workshop will work on a part of this assignment in pairs. The results will be presented and in the last part of this day we will discuss ways to judge the quality of student work on these assignments.

**Jan van Maanen: Palpable arithmetic**

Palpable arithmetic is the title of a work by the blind English mathematician Nicholas Saunderson. Saunderson (1682-1739), who was the Lucasian professor of mathematics at Cambridge, had written a manuscript 'Elements of Algebra', which was published and sold by his wife (1740). The work in two volumes contains also a biography of Saunderson and a remarkable text 'Palpable arithmetic', in which Saunderson explains a system to represent numbers with pins on a board (a type of braille for numbers, therefore, but way before Braille introduced his system). Starting from Saunderson's work, which is accessible for those who have a subscription to the English ECCO system (Eighteenth Century Collections Online), and from which a selection will be available at the Summerschool, we will discuss both the mathematical contents of the 'Elements of algebra' and the problem of how to represent mathematical text to visually impaired students. How to make mathematics palpable?

**Jo Nelissen: Mathematics and intuition**

The concept of intuition has always drawn attention and stimulated the imagination, but evoked a lot of doubt and criti-cism as well. In this article, the idea is defended that there is no point in creating a contradiction between intuition and cognition, because this contradiction is counterproductive. Intuition is considered a human activity that is closely con-nected with knowledge and that emerges during problem solving. We discern and discuss two levels of intuition, and the process of intuitive thinking is characterized in psychological terms as an immediate grasping of the meaning of a struc-ture or Gestalt. Intuition may support, as well as thwart, the learning process. In mathematics education daily life based experience (common sense) is in most cases a good starting point for the teaching process, but it also can happen that intuition - based on common sense - works towards the acquisition of mathematical insights.

The concept of intuition plays a central role in the 'intuitionism' of Brouwer. In Brouwer's view, axioms are seen as in-complete, and this means that the search for valid and true knowledge cannot be viewed as just a game of logical rules, but as a never ending human activity. Intuition can and should be supported in interactive mathematical education, be-

cause mathematics is not just a case of learning to apply standard rules and procedures. Students should be encouraged to explore unknown and unfamiliar problems. Hence, intuition is connected, not just with cognition and knowledge, but also with daring the unfamiliar, emotion and creativity.

### **Mieke Abels: The iceberg-metaphor and applications**

Researchers at the Freudenthal Institute developed the iceberg model to support teacher thinking about learning processes and strategies used by students (Boswinkel and Moerlands, 2001). This model has proved to be a powerful metaphor for illustrating how students need to experience a broad range of mathematical models to make sense of formal mathematical representations. The construction of an iceberg is a good experience to explore and discuss essential models and representations and strategies which can help students to learn mathematics with understanding. The next step is the development of a sequence of activities, based on the iceberg metaphor.

During this workshop, the participants will do several activities to develop an understanding of the iceberg metaphor, and will investigate a pathway for the concept of area.

### **Elwin Savelsbergh & Aad Goddijn: Modeling dynamic behavior; numerical and analytic approaches**

What do the temperature of your coffee and the number of rabbits in the Dutch coastal dunes have in common? At least it is this: they change over time, and the rate of change any given moment can be derived from the properties of the situation at that given moment. Now, given that information, could you predict the future development of such a system?

We just taught a novel course for Senior High School students about dynamic systems modeling, in which we combine two approaches to solve this issue: through numerical computer models and through solving differential equations. The novelty about our approach is that we teach science and mathematics views on dynamic systems in close interaction, and that numerical solution approaches are being used not only as a practical tool but also to provide a deeper insight into the mathematics of differential equations.

In this session you will get hands-on experience; and we will report about our classroom experiences. In a final example we will investigate whether a three species ecosystem could survive, or whether such a system is bound to collapse.

### **Dolly van Eerde: Language in mathematics education**

Recent studies emphasise the crucial role of language in the teaching and learning of mathematics, and the difficulties that may arise if the language aspect is neglected. Students will need to master the subject-specific language of mathematics in order to become proficient learners.

Although the growing linguistic diversity of students all over the world makes the issue of language even more urgent, all students would profit from teachers' explicit attention for language within content teaching. Hence, it is essential for every subject teacher to focus on content knowledge as well as on related language.

### **Peter Boon: The Digital Mathematics Environment (DME)**

The DME is a web-based electronic learning environment developed by the Freudenthal Institute. It provides students' access to interactive and dynamic tasks within many mathematical domains. These tasks, implemented in Java-applets, are small interactive pedagogical programs, which students can use for several purposes, such as to explore a problem situation, to discover a representation or a concept, to construct and explore 3d -

objects, to carry out a task or to practice a skill.

Embedding the applets in the DME offers several possibilities:

1. They can be organized in sequences that can be made available to groups of students or classes by the teacher.
2. The DME allows for storing the students' work with the applets.
3. The DME offers the teacher access to the students' work, by means of all-class result overviews and in detail as well.
4. The ready-made tasks in applets can be configured by the teacher. New items within the applet task can be added and existing items can, for example be adjusted to the level of the students.

A new and powerful option of the DME is a mathematical editor (authoring tool) for designing completely new tasks for students. The editor has a user-friendly formula-editor and a powerful formula parser to handle the answers of students. It offers the possibility to provide (intermediate) feedback within the solving process of the students. It is also possible to provide several tools and interactive models that support the learning process.

The DME is used by more than 100 schools in the Netherlands. The authoring tool is also used by the main textbook publishers. Development takes place in close cooperation with teachers. Meanwhile, research is conducted to theoretically underpin the value of using DME in teaching practice.

In this workshop I will show the different options and the design principles that played a role in the development of the DME and discuss them with you. In the second part of the workshop we will let you work with the DME to enable you to experience the different possibilities.

Participants of the hands-on workshop can have a look at ICT-rich, interactive learning materials and at features of the Digital Mathematics Environment, focused on space geometry. The activities will show that the applets can bridge the gap between concrete and abstract experiences.

A preview: <http://www.fi.uu.nl/dwo/en>

### **Paul Drijvers & Michiel Doorman: Supporting Functional Thinking**

In this session we will discuss key aspects of the function concept and study a recent instructional design for this topic (grade 8; 13-14 years).

The goal of the design is that students understand functions as a model for repeated calculations, representing an input-output relationship. A model that can be used to solve problems about dependency relationships with tables, graphs and formulas. The design starts with contextual problems and the focus shifts to general concepts. We will reflect on the tenets of RME (e.g. emergent modeling) with respect to this topic, the design and design research including quantitative as well as qualitative methods.

More information about the research project can be found through:

<http://www.fi.uu.nl/tooluse/en/>

### **Arthur Bakker & Adri Dierdorp: Authentic professional practices used as sources of inspiration for the education of statistics**

Authentic professional practices can be used as sources of inspiration for education. They underlying idea is that students will see the relevance of learning particular knowledge, in this case of statistics, if they see how it is used in particular professions.

For pre-university students in Grade 11 (aged 16-17) we designed educational materials based on professional practices wherein correlation and regression plays a major role.

In this workshop we show how students can be supported to reason about sampling with regard to activities that were inspired by an authentic professional practice of sport physiologists.

We will briefly show how students can use a spreadsheet to produce scatter plots, the formula for regression lines and correlation coefficients. Then, we will give a short introduction of an authentic problem, namely to determine the peak heart rate when increasing the speed on a treadmill. Finally we present a sampling task. Participants of this workshop will work on this task to judge the quality of the assignments.

### **Mieke Abels: The Assessment Pyramid**

In a balanced test, students should have the opportunity to show what they have learned and what they are able to do. Teachers may want to describe student understanding and capabilities. Also, teachers may want to describe students' misconceptions, partial understanding, etc. They want answers to questions about students' mathematical abilities, such as:

- does a student only master basic skills?
- is he able to give proper (mathematical) reasons?
- can he solve more complex problems?

Because a teacher wants to give the student feedback, (s)he needs to know which subjects are not mastered yet by the students and perhaps need more emphasis in class. It is also important to know which steps must be taken to help the least able students and give the more gifted ones enough challenging materials.

To find answers for all of these questions teachers need a great variety of tasks and tests on different levels of competency. Moreover the mathematical content of the problem and the complexity of the task must be considered.

As you see, there are many factors to be taken into account! Help is needed to bring order to the chaos. We will show a model for the classification of problems known as 'Jan de Lange's pyramid'.