summerschool UTRECHT

Summer school on Geometry
20/08/2018 – 31/08/2018

Course Leader: Dr. Johan van de Leur
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The summer school will take place in education room 6.11 on the 6th floor of the Hans Freudenthalbuilding, Budapestlaan 6, Utrecht.

Please find information about travel directions to the location at the end of the programme.

The Mathematics Department will provide lunch on Monday August 20 and Friday August 31 and dinner on Monday August 20 and Tuesday August 28. At all other times you will be expected to bring/buy your own lunch.

Week 1	
Saturday and Sunday, August 18 and 19	***Key pick-up*** You will find the exact key pick up location in the pre-departure information, which becomes available after you have paid the course fee.
Monday, August 20	***Dr. Johan van de Leur: Frieze Patterns*** Frieze patterns appear in many classical buildings such as the Alhambra in Granada. In mathematics they were introduced in the early 70's by the Canadian geometer Coxeter. They have a simple formulation and we will mainly focus on their properties. There are many connections to other areas of mathematics, such as Grassmannians, i.e., linear subspaces of n-dimensional space, and cluster algebras. One of the first connections was discovered in 1973 by Coxeter, together with Conway he found a relation to triangulations of polygons. ***10.00 – Registration and coffee*** ***11.00 – Lecture*** ***11.45 – Exercise session*** ***12.30 – Lunch at the Math Library, 7th floor***

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	13.30 – Lecture ***14.45 – Exercise session*** ***16.00 – Walk along the river the Kromme Rijn to Theehuis Rhijnauwen*** ***18.00 – Dinner at Theehuis Rhijnauwen, Rhijnauwenselaan 16, Bunnik***
Tuesday, August 21	***Dr. Jaap van Oosten: The Topology of Ordinal Numbers*** Ordinal numbers were defined by Cantor, as a way of "counting beyond infinity". They play a vital role in the foundations of mathematics, but they are also a source of examples and counterexamples in topology. We shall get to see both aspects at work, and appreciate the interplay between them. ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.30 – Lunch break*** ***13.30 – Lecture*** ***15.30 till 17.00 – Exercise session***
Wednesday, August 22	***Prof. Frans Oort: Elliptic curves and the Last Entry by Gauss*** On 9 July 1814 Gauss wrote in his Notebook a remarkable expectation. We aim to understand this claim: we need aspects of elementary algebra and number theory, of algebraic geometry and in particular properties of elliptic curves over a finite field. In this way we obtain a gentle and concrete introduction into these parts of mathematics. We will also see that this particular case studied by Gauss can be seen as a prelude to further developments: the PhD thesis by Emil Artin, formulating an analogue of the Riemann Hypothesis, culminating in conjectures by Weil: a fascinating time line in mathematics of the last two centuries. Easy exercises and concrete examples will be the backbone of this course. ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.30 – Lunch break*** ***13.30 – Lecture*** ***15.30 till 17.00 – Exercise session***
Thursday, August 23	***Dr. Gil Cavalcanti: Euler Characteristic and Classification of Surfaces*** Surfaces, such as the round sphere or the inner tube of a tyre, provide us with the first nontrivial and yet still intuitive examples of manifolds. Already here there is room for interesting behavior as displayed by the Möbius band, the Klein bottle and the mind-boggling projective space.

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	The question we want to answer is: “Is it possible to list all possible compact surfaces?” To answer this question we will need two ingredients. The first is a way to construct surfaces from simple building blocks. The second is a systematic way to tell apart nonequivalent surfaces. ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.30 – Lunch break*** ***13.30 – Lecture*** ***15.30 till 17.00 – Exercise session***
Friday, August 24	***Dr. Martijn Kool: Curves and Singularities*** A complex plane algebraic curve is the solution set of a complex polynomial in two variables. We will study singularities of such curves, i.e. points where the curve does not look “smooth”. Singularities can be resolved: think of the shadow cast by a looping of a roller coaster. I will present Enriques-Chisini’s classical method for “resolving” plane curve singularities. We will also study the topological nature of singularities by relating them to knots. This topic involves lots of hands-on explicit calculations. Time permitting, I will sketch a modern result of Oblomkov-Shende, which relates the topology of configuration spaces of points on a plane curve singularity to invariants of the corresponding knot. ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.30 – Lunch break*** ***13.30 – Lecture*** ***15.30 till 17.00 – Exercise session***

<table>
<tr><th colspan="2">Week 2</th></tr>
<tr><td>Saturday and Sunday, August 25 and 26</td><td>For the social programme organised by the university for all the summer school students, see https://www.utrechtsummerschool.nl/social-programme</td></tr>
<tr><td>Monday, August 27</td><td>Dr. Alvaro del Pino Gomez: A Gentle Introduction to Morse Theory

A smooth manifold is a (Hausdorff and second countable) topological space that locally looks like standard Euclidean space. Examples familiar to the students are, for instance, the circle (which has dimension 1) and the surfaces (like the sphere or the torus, which have dimension 2).
In a smooth manifold we can do calculus/analysis much like we do it in R^n. In particular, we can speak of a function being differentiable (to a given order) and we can study its critical points. The question at the heart of Morse theory is the following: "what can a function tell us about the topology of the manifold?".
During the course we will explore this question in the particular case of surfaces (for the sake of simplicity). In particular, we will construct an invariant of the manifold called Morse homology.

Prerequisites:
We assume that the student has taken a basic Topology course and the first year Calculus courses. A knowledge of manifold theory, simplicial/singular homology, or de Rham cohomology is potentially useful, but not necessary.
This course is a natural follow up of "Euler Characteristic and Classification of Surfaces", by Gil Cavalcanti.

References:
A standard reference for the topic (including a nice first chapter dedicated solely to surfaces) is the book:
Y. Matsumoto. An Introduction to Morse Theory. Translations of Mathematical Monographs. Iwanami Series in Modern Mathematics vol. 208 (2002)

09.00 – Lecture
11.00 – Exercise session
12.30 – Lunch break
13.30 – Lecture
15.30 till 17.00 – Exercise session</td></tr>
<tr><td>Tuesday, August 28</td><td>Marieke van der Wegen MSc: Chips Firing on Graphs

We will study multigraphs, graphs where there can be multiple parallel edges between vertices. First we will study the first Betti number of a graph, this is the number of independent cycles. This number is related to the genus of a surface. After this we will study a chip ring game on graphs.</td></tr>
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	We start with some distribution of chips over the vertices of the graph, and dene a ring rule which allows chips to move along the edges of the graph. Starting from a distribution, can we re vertices such that we obtain a distribution where every vertex has a non-negative amount of chips? ***09.30 – Class*** ***12.00 – Lunch break*** ***13.30 till 16.00 – Class*** ***18.00 – Dinner in the centre of Utrecht*** ***Explanatory parts will be alternated with working on selected exercises and projects (alone and/or in groups)***
Wednesday, August 29	***Prof. Gunther Cornelissen: Zeta Functions*** After a survey about the Riemann zeta function, we focus on three different types of zeta functions, counting solutions to equations in finite fields, counting walks on graphs and counting fixed points of dynamical systems. We consider (dis-)similarities with the case of the Riemann zeta function, such as issues of analytic continuation, poles, zeros and residues, functional equations and the Riemann hypothesis. ***09.30 – Class*** ***12.00 – Lunch break*** ***13.30 till 16.00 – Class*** ***Explanatory parts will be alternated with working on selected exercises and projects (alone and/or in groups)***
Thursday, August 30	***Prof. Erik van den Ban: Geometry and Analysis on SL(2, R)*** ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.30 – Lunch break*** ***13.30 – Lecture*** ***15.30 till 17.00 – Exercise session***
Friday, August 31	***Prof. Frits Beukers: Diophantine Equations*** Diophantine equations are equations of the form $F(x_1 \cdots x_n) = 0$ where F is a polynomial with integer coefficients and where the unknowns x_i are in the set of integers. Much is known about such equations, but even more is unknown. It is conjectured that the complex geometry of the algebraic variety given by $F = 0$ strongly influences the nature of the integer solution set. We shall illustrate these influences by

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	way of example in the form of the generalization of the Fermat equation: $x^p + y^q = z^r$ where $p, q, r \geq 2$ are given integers. We start with the well-known Pythagorean triples $x^2 + y^2 = z^2$ but very quickly increase the values of the exponents p, q, r. For the course we shall distribute course notes that go much beyond what we do in class. ***09.00 – Lecture*** ***11.00 – Exercise session*** ***12.00 – Lunch at the Math Library, 7th floor***

The summer school is located at the Hans Freudenthal Building, Budapestlaan 6 in Utrecht. You can easily bike from the centre of the city to the university. On the website of the summer schools organised by the university you can find information about renting bikes or using the public transport in Utrecht: https://www.utrechtsummerschool.nl/utrecht/getting-around .

A useful website for public transport is: http://9292.nl/en/

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